

**Position & Source:** Position vector  $\vec{r}$ , source vector  $\vec{r}'$ , separation vector  $\vec{\Delta r} = \vec{r} - \vec{r}'$

**Fundamental Theorems of Vector Calculus:**

$$\int_{\vec{a}}^{\vec{b}} \nabla f \cdot \vec{dl} = f(\vec{b}) - f(\vec{a}) \quad \int \nabla \cdot \vec{A} \, d\tau = \oint \vec{A} \cdot \vec{da} \quad \int (\nabla \times \vec{A}) \cdot \vec{da} = \oint \vec{A} \cdot \vec{dl}$$

**Cartesian Coordinates:**  $\vec{dl} = dx\hat{x} + dy\hat{y} + dz\hat{z} \quad d\tau = dx \, dy \, dz$

$$\nabla f = \frac{\partial f}{\partial x} \hat{x} + \frac{\partial f}{\partial y} \hat{y} + \frac{\partial f}{\partial z} \hat{z} \quad \nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \left( \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \hat{x} + \left( \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \hat{y} + \left( \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \hat{z} \quad \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

**Spherical Coordinates:**  $x = r \sin\theta \cos\phi, y = r \sin\theta \sin\phi, z = r \cos\theta$

$$\vec{dl} = dr\hat{r} + r \, d\theta\hat{\theta} + r \sin\theta \, d\phi\hat{\phi} \quad d\tau = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$\nabla f = \frac{\partial f}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial f}{\partial \theta} \hat{\theta} + \frac{1}{r \sin\theta} \frac{\partial f}{\partial \phi} \hat{\phi} \quad \nabla \cdot \vec{A} = \frac{1}{r^2} \frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r \sin\theta} \frac{\partial(\sin\theta A_\theta)}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{r \sin\theta} \left( \frac{\partial(\sin\theta A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi} \right) \hat{r} + \frac{1}{r} \left( \frac{1}{\sin\theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial(r A_\phi)}{\partial r} \right) \hat{\theta} + \frac{1}{r} \left( \frac{\partial(r A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right) \hat{\phi}$$

$$\nabla^2 f = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left( \sin\theta \frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 f}{\partial \phi^2}$$

**Cylindrical Coordinates:**  $x = s \cos\phi, y = s \sin\phi, z = z$

$$\vec{dl} = ds\hat{s} + s \, d\phi\hat{\phi} + dz\hat{z} \quad d\tau = s \, ds \, d\phi \, dz$$

$$\nabla f = \frac{\partial f}{\partial s} \hat{s} + \frac{1}{s} \frac{\partial f}{\partial \phi} \hat{\phi} + \frac{\partial f}{\partial z} \hat{z} \quad \nabla \cdot \vec{A} = \frac{1}{s} \frac{\partial(s A_s)}{\partial s} + \frac{1}{s} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \left( \frac{1}{s} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) \hat{s} + \left( \frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s} \right) \hat{\phi} + \frac{1}{s} \left( \frac{\partial(s A_\phi)}{\partial s} - \frac{\partial A_s}{\partial \phi} \right) \hat{z} \quad \nabla^2 f = \frac{1}{s} \frac{\partial}{\partial s} \left( s \frac{\partial f}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

**Tensor Math:**  $(\vec{a} \cdot \vec{T})_j = \sum_i a_i T_{ij} \quad (\vec{T} \cdot \vec{a})_j = \sum_i T_{ji} a_i$

**Lorentz Force:**  $\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B}), \quad \text{On Wire: } \vec{F}_{mag} = \int I(\vec{dl} \times \vec{B})$

**Maxwell's Equations:**

$$\begin{aligned} \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} & \oint \vec{E} \cdot \vec{dl} &= -\frac{d}{dt} \int \vec{B} \cdot \vec{da} \\ \nabla \cdot \vec{E} &= \rho / \epsilon_0 & \oint \vec{E} \cdot \vec{da} &= Q_{enc} / \epsilon_0 \\ \nabla \times \vec{B} &= \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} & \oint \vec{B} \cdot \vec{dl} &= \mu_0 I_{enc} + \mu_0 \epsilon_0 \frac{d}{dt} \int \vec{E} \cdot \vec{da} \\ \nabla \cdot \vec{B} &= 0 & \oint \vec{B} \cdot \vec{da} &= 0 \end{aligned}$$

**Fields in Matter:**  $\vec{P} = \vec{p}/\text{volume}$   $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$   $\sigma_b = \vec{P} \cdot \hat{n}$   $\rho_b = -\nabla \cdot \vec{P}$   $J_p = \frac{\partial \vec{P}}{\partial t}$   
 $\vec{M} = \vec{m}/\text{volume}$   $\vec{H} = \frac{1}{\mu_0} \vec{B} - \vec{M}$   $\vec{K}_b = \vec{M} \times \hat{n}$   $\vec{J}_b = \nabla \times \vec{M}$   
 $\nabla \cdot \vec{D} = \rho_f$   $\oint \vec{D} \cdot \vec{d}\vec{a} = Q_{f\_enc}$   
 $\nabla \times \vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t}$   $\oint \vec{H} \cdot \vec{d}\vec{l} = I_{f\_enc} + \frac{d}{dt} \int \vec{D} \cdot \vec{d}\vec{a}$

**Linear Materials:**  $\vec{P} = \epsilon_0 \chi_e \vec{E}$   $\vec{D} = \epsilon \vec{E} = (1 + \chi_e) \epsilon_0 \vec{E} = \epsilon_r \epsilon_0 \vec{E}$   
 $\vec{M} = \chi_m \vec{H}$   $\vec{B} = \mu \vec{H} = (1 + \chi_m) \mu_0 \vec{H}$

**Boundary Conditions:**  $\Delta D_{\perp} = \sigma_f$   $\Delta \vec{E}_{\parallel} = 0$   $\Delta \vec{D}_{\parallel} = \Delta \vec{P}_{\parallel}$   
 $\Delta \vec{H}_{\parallel} = \vec{K}_f \times \hat{n}$   $\Delta B_{\perp} = 0$   $\Delta H_{\perp} = -\Delta M_{\perp}$

**Ohm's Law and EMF:**  $\vec{J} = \sigma \vec{E}$   $\mathcal{E} = \oint (\vec{F}/q) \cdot \vec{d}\vec{l}$   $\mathcal{E}_{\text{motional}} = -\frac{d\Phi_B}{dt}$

**Inductance:**  $M = \frac{\Phi_{B2}}{I_1} = \frac{\Phi_{B1}}{I_2}$   $L = \frac{\Phi_{B1}}{I_1}$   $\mathcal{E}_{\text{induced}} = -L \frac{dI}{dt}$   $\frac{dW}{dt} = \mathcal{E}I$

**Continuity of Charge/Current:**  $\frac{\partial \rho}{\partial t} = -\nabla \cdot \vec{J}$   $\frac{\partial \rho_b}{\partial t} = -\nabla \cdot \vec{J}_p$

**Energy & Momentum:**  $\vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B})$   $u_{EM} = \frac{1}{2} (\epsilon_0 E^2 + \frac{B^2}{\mu_0})$   $U_{EM} = \int u_{EM} dt$   
 $T_{ij} = \epsilon_0 (E_i E_j - \frac{1}{2} \delta_{ij} E^2) + \frac{1}{\mu_0} (B_i B_j - \frac{1}{2} \delta_{ij} B^2)$   $\vec{g} = \epsilon_0 (\vec{E} \times \vec{B}) = \mu_0 \epsilon_0 \vec{S}$   $\vec{p}_{EM} = \int \vec{g} dt$   
 $\frac{dW}{dt} = -\oint \vec{S} \cdot \vec{d}\vec{a} - \frac{dU_{EM}}{dt}$   $\frac{\partial u_{EM}}{\partial t} = -\nabla \cdot \vec{S}$  if  $\frac{dW}{dt} = 0$   
 $\vec{F} = \frac{d\vec{p}_{mech}}{dt} = \oint \vec{T} \cdot \vec{d}\vec{a} - \frac{d\vec{p}_{EM}}{dt}$   $\vec{f} = \nabla \cdot \vec{T} - \frac{\partial \vec{g}}{\partial t}$   $\frac{\partial \vec{g}}{\partial t} = \nabla \cdot \vec{T}$  if  $\vec{f} = 0$

**Linear materials:**  $\vec{S} = \frac{1}{\mu} (\vec{E} \times \vec{B})$   $u_{EM} = \frac{1}{2} (\epsilon E^2 + \frac{B^2}{\mu})$

### EM Plane Waves:

**Complex:**  $\vec{E}(\vec{r}, t) = \vec{E}_0 \exp(i(\vec{k} \cdot \vec{r} - \omega t)) \hat{n}$   $\vec{B}(\vec{r}, t) = \frac{k}{\omega} \vec{E}_0 \exp(i(\vec{k} \cdot \vec{r} - \omega t)) (\hat{k} \times \hat{n})$   
**Real:**  $\vec{E}(\vec{r}, t) = E_0 \cos(\vec{k} \cdot \vec{r} - \omega t + \delta) \hat{n}$   $\vec{B}(\vec{r}, t) = \frac{k}{\omega} E_0 \cos(\vec{k} \cdot \vec{r} - \omega t + \delta) (\hat{k} \times \hat{n})$   
 $\frac{\omega}{k} = c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$   $\langle u \rangle = \frac{1}{2} \epsilon_0 E_0^2$   $\langle \vec{S} \rangle = c \langle u \rangle \hat{k} = I \hat{k} = \frac{1}{2} c \epsilon_0 E_0^2 \hat{k}$   $\langle \vec{g} \rangle = \frac{\langle \vec{S} \rangle}{c^2} = \frac{1}{2c} \epsilon_0 E_0^2 \hat{k}$

**Linear materials:**  $\frac{\omega}{k} = v = \frac{1}{\sqrt{\mu \epsilon}}$   $\langle \vec{S} \rangle = v \langle u \rangle \hat{k} = I \hat{k} = \frac{1}{2} v \epsilon E_0^2 \hat{k}$   $n = \frac{c}{v} = \sqrt{\frac{\mu \epsilon}{\mu_0 \epsilon_0}}$

**Conductors:**  $k_r = \omega \sqrt{\frac{\mu \epsilon}{2} [\sqrt{1 + (\sigma/\epsilon \omega)^2} + 1]}^{1/2}$   $k_i = \omega \sqrt{\frac{\mu \epsilon}{2} [\sqrt{1 + (\sigma/\epsilon \omega)^2} - 1]}^{1/2}$

$\tilde{k} = k_r + ik_i$   $d = 1/k_i$   $n = \frac{c}{v} = \frac{ck_r}{\omega}$   $\frac{B_0}{E_0} = \frac{|k|}{\omega} = \frac{\sqrt{k_r^2 + k_i^2}}{\omega}$

**Reflection/Transmission:**  $\theta_I = \theta_R$   $\frac{\sin \theta_T}{\sin \theta_I} = \frac{n_1}{n_2}$   $\alpha = \frac{\cos \theta_T}{\cos \theta_I}$   $\beta = \frac{\mu_1 n_2}{\mu_2 n_1} = \frac{\mu_1 v_1}{\mu_2 v_2}$

$\vec{E}_{OR} = \frac{\alpha - \beta}{\alpha + \beta} \vec{E}_{OI}$   $\vec{E}_{OT} = \frac{2}{\alpha + \beta} \vec{E}_{OI}$   $R = \langle S_R \rangle / \langle S_I \rangle$   $T = \langle S_T \rangle \cos \theta_T / \langle S_I \rangle \cos \theta_I$   $R + T = 1$

**Potentials:**  $\vec{B} = \nabla \times \vec{A}$   $\vec{E} = -\nabla V - \frac{\partial \vec{A}}{\partial t}$  Gauge Transformation:  $\vec{A}' = \vec{A} + \nabla \lambda$ ,  $V' = V - \frac{\partial \lambda}{\partial t}$

$$\nabla^2 V + \frac{\partial}{\partial t} (\nabla \cdot \vec{A}) = -\frac{\rho}{\epsilon_0} \quad (\nabla^2 \vec{A} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2}) - \nabla (\nabla \cdot \vec{A} + \mu_0 \epsilon_0 \frac{\partial V}{\partial t}) = -\mu_0 \vec{J}$$

Coulomb Gauge:  $\nabla \cdot \vec{A} = 0 \Rightarrow \nabla^2 V = -\frac{\rho}{\epsilon_0} \quad (\nabla^2 \vec{A} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2}) - \mu_0 \epsilon_0 \nabla \left( \frac{\partial V}{\partial t} \right) = -\mu_0 \vec{J}$

Lorenz Gauge:  $\nabla \cdot \vec{A} = -\mu_0 \epsilon_0 \frac{\partial V}{\partial t} \Rightarrow \nabla^2 V - \mu_0 \epsilon_0 \frac{\partial^2 V}{\partial t^2} = -\frac{\rho}{\epsilon_0} \quad (\nabla^2 \vec{A} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{A}}{\partial t^2}) = -\mu_0 \vec{J}$

**Retarded Potentials:**  $t_r = t - \frac{\Delta r}{c}$   $V(\vec{r}, t) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}', t_r)}{\Delta r} d\tau'$   $\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\vec{J}(\vec{r}', t_r)}{\Delta r} d\tau'$

**Liénard-Wiechert Potentials:**  $V(\vec{r}, t) = \frac{1}{4\pi\epsilon_0} \frac{qc}{(c\Delta r - \Delta\vec{r} \cdot \vec{v})}$   $\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi} \frac{qc\vec{v}}{(c\Delta r - \Delta\vec{r} \cdot \vec{v})} = \frac{\vec{v}}{c^2} V(\vec{r}, t)$

$$\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{\Delta r}{(\Delta\vec{r} \cdot \vec{u})^3} [(c^2 - v^2)\vec{u} + \Delta\vec{r} \times (\vec{u} \times \vec{a})] \quad \vec{B}(\vec{r}, t) = \frac{1}{c} \Delta\vec{r} \times \vec{E} \quad \vec{u} = c\Delta\vec{r} - \vec{v}$$

Constant Velocity:  $\vec{E}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{1-v^2/c^2}{(1-v^2 \sin^2 \theta / c^2)^{3/2}} \frac{\hat{R}}{R^2}$   $\vec{B}(\vec{r}, t) = \frac{1}{c} \Delta\vec{r} \times \vec{E} = \frac{1}{c^2} \vec{v} \times \vec{E}$

**Radiation: Electric Dipole:**  $\vec{S}(\vec{r}, t) \cong \frac{\mu_0}{c} \left\{ \frac{p_0 \omega^2 \sin \theta}{4\pi r} \cos \left[ \omega \left( t - \frac{r}{c} \right) \right] \right\}^2 \hat{r}$   $P = \frac{\mu_0 p_0^2 \omega^4}{12\pi c}$

Magnetic Dipole:  $\vec{S}(\vec{r}, t) \cong \frac{\mu_0}{c} \left\{ \frac{m_0 \omega^2 \sin \theta}{4\pi c r} \cos \left[ \omega \left( t - \frac{r}{c} \right) \right] \right\}^2 \hat{r}$   $P = \frac{\mu_0 m_0^2 \omega^4}{12\pi c^3}$

Multipole:  $\vec{E}(\vec{r}, t) \cong \frac{\mu_0}{4\pi r} [\hat{r} \times (\hat{r} \times \ddot{\vec{p}})]$   $\vec{B}(\vec{r}, t) \cong -\frac{\mu_0}{4\pi r c} [\hat{r} \times \ddot{\vec{p}}]$   $\vec{S}(\vec{r}, t) \cong \frac{\mu_0 \ddot{p}^2 \sin^2 \theta}{16\pi^2 c r^2} \hat{r}$

Pt. Chrg:  $\vec{E}_{rad}(\vec{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{\Delta r}{(\Delta\vec{r} \cdot \vec{u})^3} [\Delta\vec{r} \times (\vec{u} \times \vec{a})]$   $\vec{S}_{rad} = \frac{1}{\mu_0 c} E_{rad}^2 \Delta\vec{r}$   $P = \frac{\mu_0 q^2 \gamma^6}{6\pi c} \left( a^2 - \left| \frac{\vec{v} \times \vec{a}}{c} \right|^2 \right)$

Low Velocity:  $\vec{S}_{rad} = \frac{\mu_0 q^2 a^2 \sin^2 \theta}{16\pi^2 c} \frac{\Delta\vec{r}}{\Delta r^2}$   $P = \oint \vec{S} \cdot \vec{d}\vec{a} = \frac{\mu_0 q^2 a^2}{6\pi c}$